

Improper Integrals

Example

1. Let $\alpha \in \mathbb{R}$. Calculate $\int_1^{\infty} \frac{1}{x^\alpha} dx$.

Problems

2. True False It is possible for the integral $\int_1^{\infty} f(x)$ to be neither a finite number nor infinity.
3. True False By the above example, we know that $\int_0^{\infty} \frac{1}{x^3} dx$ converges.
4. True False If $\lim_{x \rightarrow \infty} f(x) = 0$, then $\int_1^{\infty} f(x) dx$ converges.
5. Calculate $\int_3^{\infty} \frac{1}{x \ln(x)}$.
6. Calculate $\int_1^{\infty} e^{-5x} dx$.
7. Calculate $\int_1^{\infty} \frac{x}{\sqrt{x^2 + 1}} dx$.
8. Calculate $\int_0^{\infty} \frac{1}{1 + x^2} dx$.
9. Calculate $\int_1^{\infty} x e^{-2x} dx$.
10. Calculate $\int_1^{\infty} \frac{2x}{1 + x^2} dx$.

Convergent/Divergent Integrals

Example

11. Does $\int_0^{\infty} \frac{\arctan^2(x)}{\sqrt{1 + x^4}} dx$ converge?

Problems

12. True False If $a < b$ then $ac < bc$.

13. True False If $a < b$, then $\frac{1}{a} > \frac{1}{b}$.

14. True False If we can find a function g such that $0 \leq f \leq g$, then $\int_1^\infty f(x)dx$ converges.

15. Does $\int_3^\infty \frac{1}{\sqrt{x} \ln(x)}$ converge?

16. Does $\int_1^\infty e^{-5x\sqrt{x}} dx$ converge?

17. Does $\int_1^\infty \frac{x}{\sqrt{x^2+1} - e^{-x}} dx$ converge?

18. Does $\int_0^\infty \frac{1}{(1+x^2)^2} dx$ converge?

19. Does $\int_1^\infty \sqrt{x}e^{-2x}$ converge?

20. Does $\int_1^\infty \frac{2x + 2xe^{-x}}{1+x^2} dx$ converge?